Quality Control &
Statistical Process Control

1- Statistical Process Control

* A Process used to monitor standards by talling measurements and Corrective action as a product or service being produced.
* All processes are subject to a certain degree of variability
* There are two types of variations:
  1. Natural variations [Natural Causes]: Variability that affects every production process to some degree and is to be expected
  2. Assignable variation: variation in a production process that can be traced to specific causes [such as machine wear, misadjusted equipment or untrained worlers]

A process is said to be operating in <u>Statistical Control</u> when the only source of variation is Natural Variation.

* How to test a process?
  1. Detect & Eliminate special or assignable causes of variation
  2. Predict the process performance
  3. Assess customer expectations.

2. Control Charts:

* A graphical presentation of process data over time

Control charts

For variables

* examples are weight, speed, length, strength, etc.

$\bar{X}$-chart | R-chart

For Attributes

* typically clasified as defective or nondefective.
* Measuring defectives involves counting them.

P-chart | C-chart

3. Control charts for variables:

1. $\bar{X}$-Chart: a quality control chart for variables that indicates when changes occur in the central tendency [the mean] of a production process.

→ These changes might be due to such factors as tool wear, a gradual increase in temperature, etc.

* $\bar{X}$-chart limits approaches:

① $UCL = \bar{\bar{X}} + Z\sigma_{\bar{X}}$, where $\bar{\bar{X}}$ = Mean of the sample means

$LCL = \bar{\bar{X}} - Z\sigma_{\bar{X}}$

Z = # of normal standard deviations [2 for 95.45% confidence, 3 for 99.73%]

$\sigma_{\bar{X}}$ = standard deviation of the sample means $= \sigma/\sqrt{n}$

σ = Population [process] standard deviation

n = Sample size.

Problem 1:

Given: * 99.73% Confidence $\Longrightarrow Z = 3$

* $n = 9$ [sample size]

* $\bar{\bar{X}}$ = mean of the sample means

$$= \frac{16.1 + 16.8 + 15.5 + 16.5 + 16.5 + 16.4 + 15.2 + 16.4 + 16.3 + 14.8 + 14.2 + 17.3}{12}$$

$$= \frac{192}{12} = 16$$

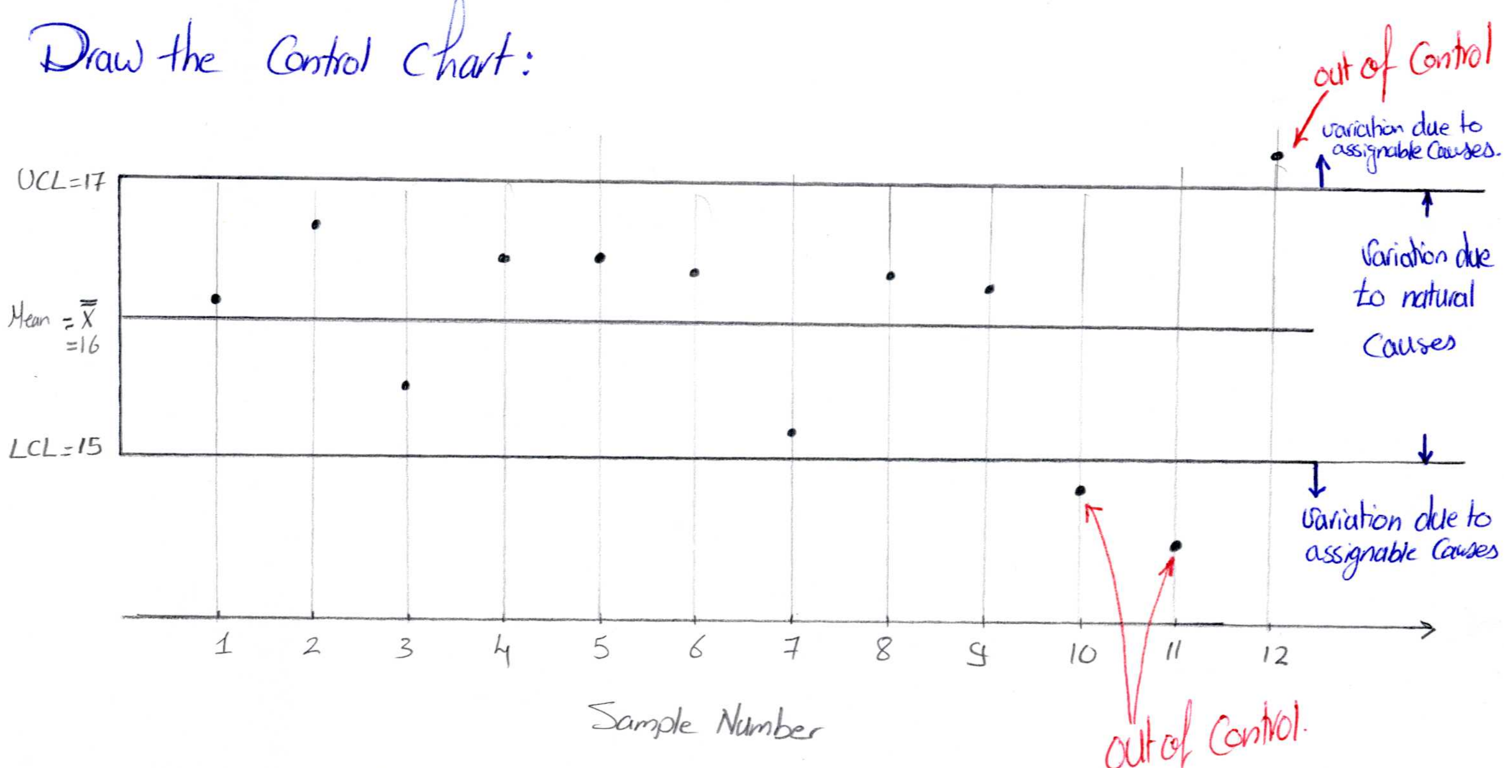
* σ = Population st. dev. = 1

$$\therefore \sigma_{\bar{X}} = \sigma / \sqrt{n} = 1/\sqrt{9} = \frac{1}{3}$$

$$\therefore UCL_{\bar{X}} = \bar{\bar{X}} + Z\sigma_{\bar{X}} = 16 + 3 * \frac{1}{3} = 17 \text{ ounces}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - Z\sigma_{\bar{X}} = 16 - 3 * \frac{1}{3} = 15 \text{ ounces.}$$

* Draw the Control Chart:



Because the means of recent sample averages fall outside the upper and lower Control limits of 17 & 15, we Can Conclude that the Process is not in Control.

→ In Case no information is given about the population standard deviation use these formulas:

$$UCL_{\bar{x}} = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{X}} - A_2 \bar{R}$$

where, $\bar{R}$ = average range of the samples

A_2 = value found in table [Depends on the sample size]

* Range: The difference between the largest & smallest items in one sample.

2- R-Charts:

* A Control chart that tracks the "range" within a sample; it indicates that a gain or loss in uniformity has occured in dispersion of a production process.

$$UCL_R = D_4 \bar{R}$$

$$LCL_R = D_3 \bar{R}$$

where, UCL_R = upper control chart limit for the range

LCL_R = lower " " " " "

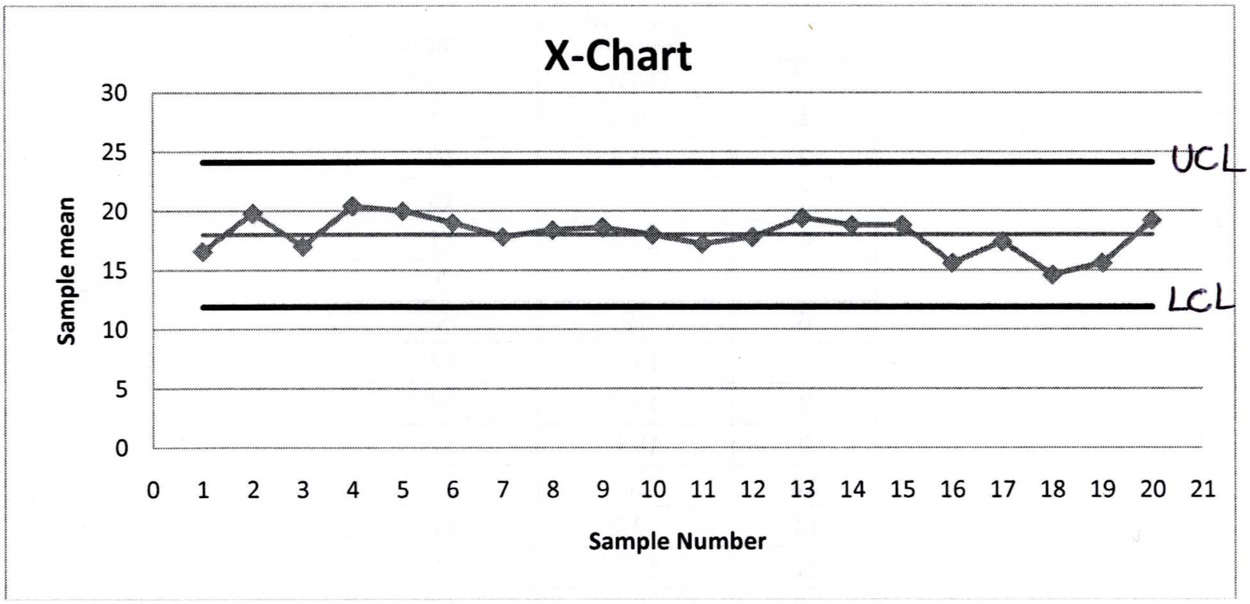
D_4 & D_3 = values from given table.

Question [redacted]2

sample	Sample Mean	Sample Range
1	16.6	11
2	19.8	10
3	17	15
4	20.4	11
5	20	9
6	19	5
7	17.8	14
8	18.4	14
9	18.6	12
10	18	11
11	17.2	12
12	17.8	6
13	19.4	13
14	18.8	11
15	18.8	8
16	15.6	12
17	17.4	10
18	14.6	6
19	15.6	10
20	19.2	11
Σ	**360**	**211**

a) X- Chart:

- Grand mean= $\frac{\bar{X}_1+\bar{X}_2+\bar{X}_3+\cdots}{number\ of\ samples}$ = 360/20 = 18
- Mean range = $\frac{\bar{R}_1+\bar{R}_2+\bar{R}_3+\cdots}{number\ of\ samples}$ = 211/20= 10.55
- Control limits:
  - UCL= Grand Mean + A_2 *Mean range, where A_2=0.58 [n=5]
    UCL= 18+0.58*10.55= 24.119
  - LCL= Grand Mean - A_2 *Mean range
    LCL= 18-0.58*10.55=11.881



- According to the X-chart the process appears to be in control because all the sample means are within the UCL and LCL.

Problem 3:

R-Chart:

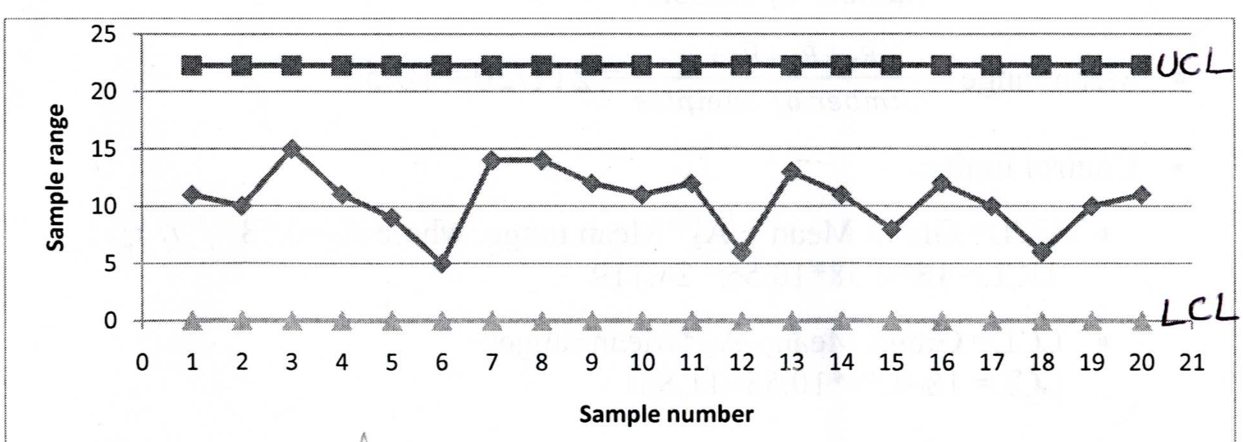
- $UCL_R = D_4 *$ Mean range, where $D_4 = 2.11$

  $UCL_R = 2.11 * 10.55$

  $UCL_R = 22.2605$

- $LCL_R = D_3 *$ Mean range, where $D_3 = 0$

  $LCL_R = 0$



According to this chart all the sample ranges are within the UCL & LCL, therefore the process is in Control.