# Linear Programming

1- **linear Programming:** is a mathematical technique designed to help operat
managers plan & make decisions relative to the tradeoffs necessar
to allocate resources.

For example: Selecting the product mix in a factory to make bes
use of machine- or (labor hours) while maximizing profits

## 2- All LP problems have 4 properties in Common:-

① Objective Function: a mathematical expression in linear Programming that
maximizes or minimizes some quantity [often Profit or Cost]

Maximize

For example:

Product ① makes Profit of $10/unit

Product ② makes profit of $15/unit

objective Function:- Max. : $10X_1 + 15X_2$

where, $X_1$ = Number of units that should be produced of product 1

$X_2$ = Number of units that should be Produced of product ②

Minimize

Example:

Product ① costs $50/unit to produced

Product ② costs $100/unit to be produced

Objective Function: Min $50X_1 + 10$

where $X_1$ = # of units of product ① that should be produced

$X_2$ = # of units of produ that should be produce

② Constraints: Restrictions that limit the degree to which
a manager can pursue an objective.

* less than or equal (≤): implies an upper limit on the amount of som resources
* greater than or equal (≥): specifies a min. that must be achieved
* Equal (=): specifies exactly what a mathematical expression of decision variables should equal.

③ There must be alternative courses of action to choose from

④ The objective & Constraints in LP must be expressed in terms of linear equations & (or) inequalities.

## 3- Formulating Linear Programming Problems:

Step ①: Identify whether the objective function is to Max. or Min.

step ②: Organize your given information in a table & introduce the decision variables.

| Variables | $X_1$ | $X_2$ | Available hours (units) of variable |
|---|---|---|---|
| For example: labour time | — hrs | — hrs | 200 means that there are 200 labor hours available to be used in producing $X_1, X_2$ |
| assembly time | | | |
| Painting | | | |

Step ③: Develop the mathematical relationships to describe the Constraints

# LINEAR PROGRAMMING

**Problem 1**: LP Formulation and Graphical Solution:

A small construction firm specializes in building and selling single-family homes. The firm offers two basic types of houses, model A and model B. Model A houses require 4,000 labour hours, 2 tonnes of stone, and 2,000 board feet of lumber. Model B houses require 10,000 labour hours, 3 tonnes of stone, and 2,000 board feet of lumber. Due to long lead times for ordering supplies and the scarcity of skilled and semiskilled workers in the area, the firm will be forced to rely on its present resources for the upcoming building season. It has 400,000 hours of labour, 150 tonnes of stone, and 200,000 board feet of lumber. Model A yields a profit of $1,000 per unit and model B yields $2,000per unit? Assume that the firm will be able to sell all the units it builds.

*a.* Formulate the objective function and constraints

*b.* Graph the constraints and objective function, and identify the optimum corner point

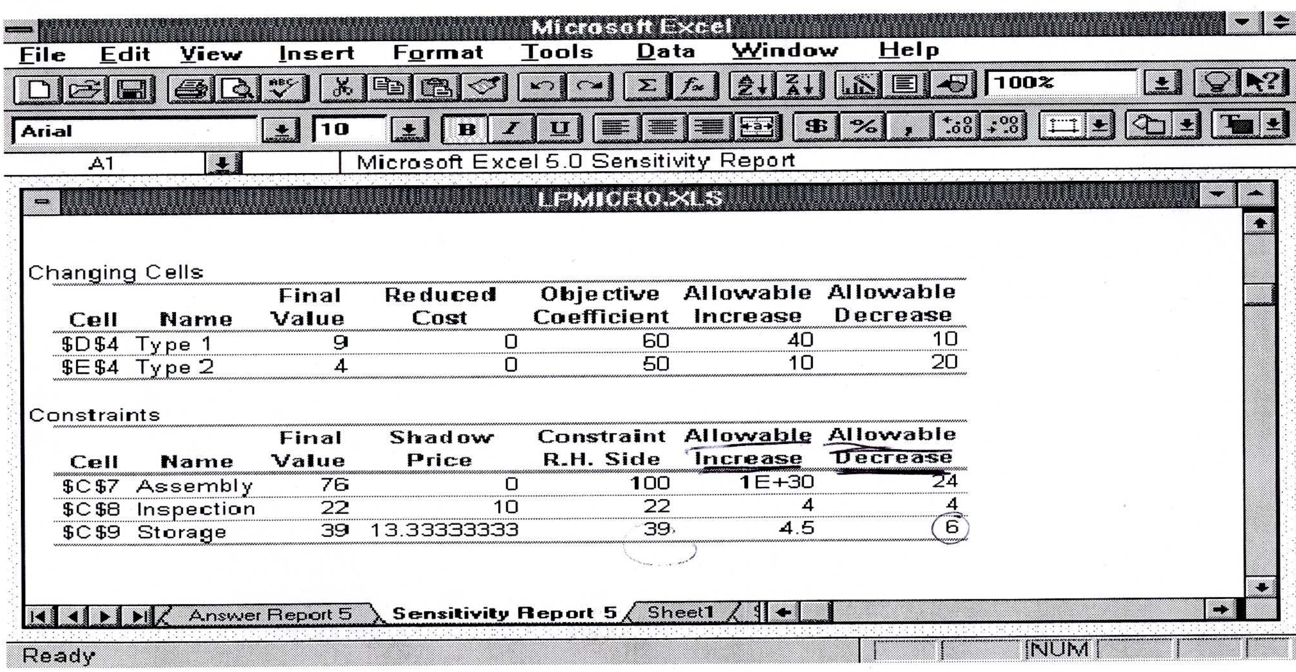
*c.* Determine the optimal quantities of models A and B, and compute the resulting profit.

**Problem 2:** Sensitivity Analysis:

Maximize $60x_1 + 50x_2$ where $x_1$ = the number of type 1 computers
$x_2$ = the number of type 2 computers

Subject to

| | | |
|---|---|---|
| Assembly | $4x_1 + 10x_2 \le$ | 100 hours |
| Inspection | $2x_1 + 1x_2 \le$ | 22 hours |
| Storage | $3x_1 + 3x_2 \le$ | 39 cubic feet |
| | $x_1, x_2 \ge 0$ | |




Changing Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $D$4 | Type 1 | 9 | 0 | 60 | 40 | 10 |
| $E$4 | Type 2 | 4 | 0 | 50 | 10 | 20 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $C$7 | Assembly | 76 | 0 | 100 | 1E+30 | 24 |
| $C$8 | Inspection | 22 | 10 | 22 | 4 | 4 |
| $C$9 | Storage | 39 | 13.33333333 | 39 | 4.5 | 6 |

# Linear Programming Problems

1- a)

| | Model A [$x_1$] | Model B [$x_2$] | Available resources |
|---|---|---|---|
| labour hours | 4000 hrs | 10,000 hrs. | 400,000 hrs |
| stones needed | 2 tonnes | 3 tonnes | 150 tonnes |
| lumber needed | 2000 feet | 2000 feet | 200,000 feet |
| Profit /unit | \$1000 | \$2000 | |

* A = Number of units of Model A that should be build

  B = " " " " Model B " " " "

* Max : $Z = 1000A + 2000B$

  subject to,

  labour : $4000A + 10{,}000B \leq 400{,}000$

  stone : $2A + 3B \leq 150$

  lumber : $2000A + 2000B \leq 200{,}000$

  $A, B \geq 0$