

Minimization Problems

Question 1:

Cohen chemicals, Inc. Produces 2 types of photo developing fluids. The first, a black and white picture chemical, costs Cohen \$2,500 per ton to produce. The second, a color photo chemical, costs \$3,000 per ton.

Based on an analysis of the current inventory levels and outstanding orders, Cohen's production manager has specified that at least 30 tons of the black and white chemical and at least 20 tons of the color chemical must be produced during the next month. In addition, the manager notes that an existing inventory of highly perishable raw material needed in both chemicals must be used within 30 days. To avoid wasting the expensive raw material, Cohen must produce a total of at least 60 tons of the photo chemicals in the next month.

- a- Formulate the objective functions and constraints.
- b- Graph the constraints and objective function and identify the optimum corner point.
- c- Determine the optimal quantities of the 2 types and compute the resulting cost.

Question 2:

Solve the following problem using graphical approach.

$$\begin{array}{ll} \text{Minimize} & Z = 8x_1 + 12x_2 \\ \text{Subject to} & 5x_1 + 2x_2 \geq 20 \\ & 4x_1 + 3x_2 \geq 24 \\ & x_2 \geq 2 \\ & x_1, x_2 \geq 0 \end{array}$$

Minimization Problems

Question 1:

- * X_1 = number of tons of black & white picture chemical produced.
- * X_2 = " " " " color picture chemical produced.

a) Objective function: Min. : $2500X_1 + 3000X_2$

subject to : $X_1 \geq 30$ tons

$X_2 \geq 20$ tons

$X_1 + X_2 \geq 60$ tons

$X_1, X_2 \geq 0$ [Nonnegativity requirement]

b) a) Corner a:

$X_2 = 20$

$\therefore X_1 + X_2 = 60 \Rightarrow X_1 = 40$

$Z = 2500 \times 40 + 3000 \times 20 = \$160,000$

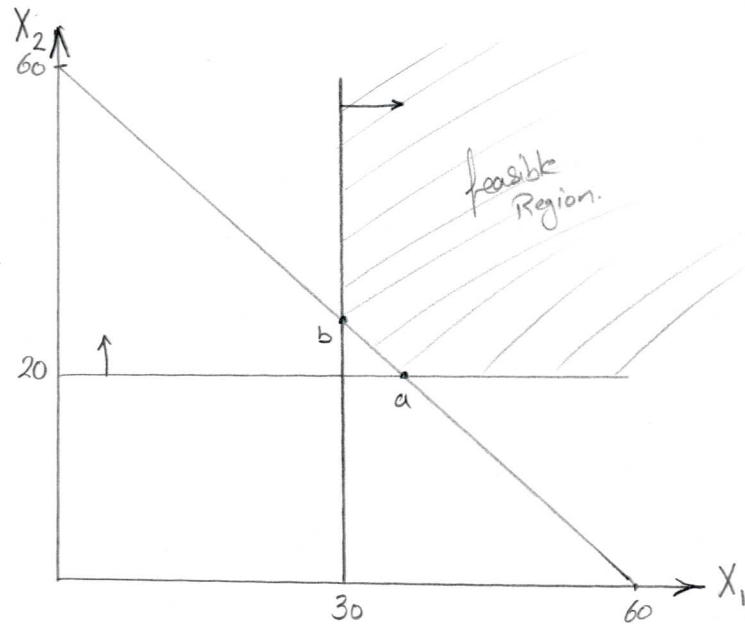
a) Corner b:

$X_1 = 30$

$\therefore X_1 + X_2 = 60 \Rightarrow X_2 = 30$

$Z = 2500 \times 30 + 3000 \times 30 = \$165,000$

\therefore Corner a is the optimal solution, since $X_1 = 40, X_2 = 20$ gives less cost.



Question 2: Min: $Z = 8X_1 + 12X_2$

subject to: ① $5X_1 + 2X_2 \geq 20$

② $4X_1 + 3X_2 \geq 24$

③ $X_2 \geq 2$

④ $X_1, X_2 \geq 0$

* for Constraint ①:

$$5X_1 + 2X_2 = 20$$

$$\text{if } X_1 = 0 \Rightarrow X_2 = 10$$

$$\text{if } X_2 = 0 \Rightarrow X_1 = 4$$

* For Constraint ②:

$$4X_1 + 3X_2 = 24$$

$$\text{if } X_1 = 0 \Rightarrow X_2 = 8$$

$$\text{if } X_2 = 0 \Rightarrow X_1 = 6$$

* Draw objective function by assuming Cost

$$\text{Assume } Z = 96$$

$$\therefore 96 = 8X_1 + 12X_2$$

$$\text{if } X_1 = 0 \Rightarrow X_2 = 8$$

$$\text{if } X_2 = 0 \Rightarrow X_1 = 12$$

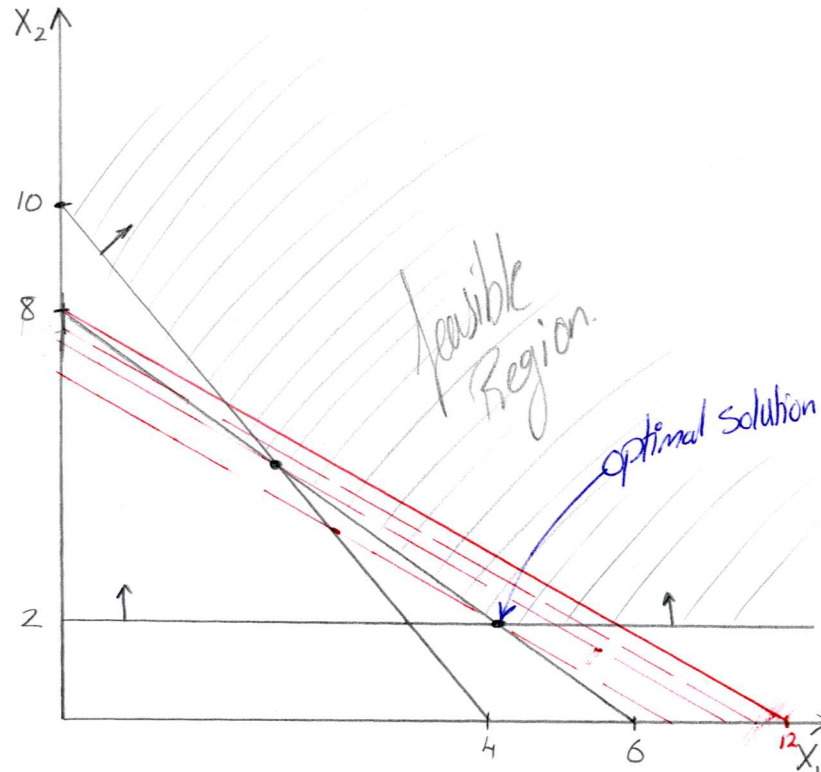
* Optimal solution: $X_2 = 2$

$$4X_1 + 3X_2 = 24$$

$$\therefore 4X_1 = 24 - 6 = 18$$

$$\therefore X_1 = 4.5$$

* Min. Cost = $8 \times 4.5 + 12 \times 2 = \60



COMM 225: Production and Operations Management
Section A, Fall 2009

LINEAR PROGRAMMING – SENSITIVITY ANALYSIS

Question 1: Kennytrail Amusement park is trying to divide its new 50 acre park into three categories: rides, food, and shops. Each acre used for rides generates \$150/hour profit; each acre used for food generates \$200/hour profit. Shops generate \$300/hour profit. There are a number of restrictions on how the space can be divided:

1. Only 10 acres of land is suitable for shops.
2. Zoning regulations require at least 1000 trees in the park. A food acre has 30 trees; a ride acre has 20 trees; while a shop acre has no trees.
3. No more than 200 people can work in the park. It takes 3 people to work an acre of rides, 6 to work an acre of food, and 5 to work an acre of shops.

The resulting linear program model is as follows:

$$\begin{array}{ll}
 \text{MAX} & 150 \text{ RIDE} + 200 \text{ FOOD} + 300 \text{ SHOP} \\
 \text{SUBJECT TO} & \\
 & \text{RIDE} + \text{FOOD} + \text{SHOP} \leq 50 \quad (\text{total}) \\
 & \text{SHOP} \leq 10 \quad (\text{requirement 1}) \\
 & 20 \text{ RIDE} + 30 \text{ FOOD} \geq 1000 \quad (\text{requirement 2}) \\
 & 03 \text{ RIDE} + 6 \text{ FOOD} + 5 \text{ SHOP} \leq 200 \quad (\text{requirement 3})
 \end{array}$$

- (a) What is the optimal allocation of the space? What is the profit/hour of the park? **(1 mark)**
- (b) City Council wants to increase our tree requirement to 1020. How much will that cost us (in \$/hour).? **(1 mark)**
- (c) Suppose the City Council wants to increase our tree requirement to 1200. How much will that cost us (in \$/hour).? **(1 mark)**
- (d) A construction firm is willing to convert 5 acres of land to make it suitable for shops. How much should Kennytrail be willing to pay for this conversion (in \$/hour). **(1 mark)**
- (e) Suppose Food only made a profit of \$180/hour. What would be the optimal allocation of the park, and what would be the profit/hour of the park? **(1 mark)**
- (f) An adjacent parcel of land has become available. It is five acres in size. The owner wants to share in our profits. How much \$/hour is Kennytrail willing to pay? **(1 mark)**
- (g) The management of Kennytrail Amusement park is considering adding a “torpical zoo” in the par. An arce of tropical zoo will have 20 trees and it would take 5 people to work on an acre of zoo. How much profit should this new theme be generating so that it is worth for the management to consider it pursuing? **(1 mark)**

